*“We now know for certain that the magic number is 20.”*

* Morley Davidson

Algorithms play a big part in the Rubik’s cube, as you saw earlier we can use algorithms with notation to describe moves on the cube when solving , but they are also used by computers, this chapter will take a look at the main way algorithms have been used by computers in relation to the cube since its invention.

One of the biggest challenges faced by mathematicians interested in the Rubik’s cube was the idea of god’s number, the maximum number of turns it would take god to solve the cube (supposing he could do it in the fewest number every time) . It turns out that number is 20. However simple this number is it took 29 years for mathematicians to pinpoint it.

The hunt for gods number was started in 1981 by British mathematician Morwen Thistlethwaite who set the upper bound of 52 moves and a lower bound of 18, he achieved these numbers using a computer algorithm designed by himself to solve the cube in (for it’s time) a remarkably small number of moves. His algorithm consisted of 4 stages , each stage reducing the possible number of combinations down from the original number of 4.33 down to only 663,552 positions, whilst this may still seem a massive number by doing these steps he has reduced the amount of work by a factor of 6.5.

The basis for this comes from, as all great things do, group theory. When you have a scrambled cube and allow any turn you are in the group G0 as explained in the group theory chapter this is the group where any position is possible so this has the usual 4.33 possible combinations. The first stage in eliminating some of these in the Thistlethwaite algorithm is to flip the edges the right way , what this means is that you would now be able to move them into position without the need to orient them at all. To be able to carry out this step you must be able to carry out quarter turns of the U and D layers , without these it’s impossible to orient the edges but once they’re all done, we no longer need to do quarter turns of U and D so we can restrict ourself to only half turns on these faces. This then puts us into a subgroup ( I really hope you paid attention in the group theory chapter) which we can call G1 , we define G1 as being the group of combinations possible by only using { L, F, R, B, D2, U2} this group then only has 2.11 positions. First stage done.

I wont bore you by going through every single stage but below I’ve summarized them into a table of the groups they achieve , the number of combinations possible and the physical manipulation needed to get to the next stage.

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| --- | --- | --- | --- |
| Stage | Group | Positions | To get to the next step |
| 0 | G0{ L, R, U, D, F, B} | 4.33 | Orient all the edges |
| 1 | G1{ L, R, U2, D2, F, B} | 2.11 | Orient all corners |
| 2 | G2{ L, R, U2, D2, F2, B2} | 1.95 | Partially position pieces |
| 3 | G3{ L2, R2, U2, D2, F2, B2} | 6.63 | Complete positioning pieces |
| 4 | G4 | 1 |  |

So that’s the theory behind it, but the challenging part is putting it into practice, at this time computers weren’t quite capable to be figuring out for themselves the moves necessary to get between stages and so the method of doing this was still very rudimentary, look up tables. Just like logarithm lookup tables through multiple values you can find the completed set of moves needed. By using some maths a bit beyond the scope of this book Thistlethwaite was able to keep the size of these lookup tables to a minimum, 21 pages in total.

By splitting up the solving into very small steps like this you can easily start to calculate the maximum numbers of moves for each step, rather than the cube as a whole. Using this Thistlethwaite managed to set an upperbound of 52 moves, but that wasn’t the end for the Thwistlethwaite algorithm. At this time Thistlethwaite was teaching at the Polytechnic of the South Bank in London where he set the problem to his students who managed to reduce the upperbound to 50 moves by very comprehensive analysis. Over time using the algorithm the best upperbound achieved was 45 moves.

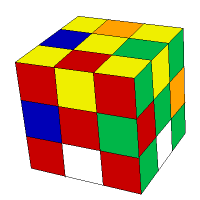
After this variations of the algorithm started popping up, optimizing each stage, the next most notable being Kociembas two phase algorithm , which American mathematician Michael Reid managed to use to set an upperbound of 28 moves, Kociemba is effectively just a variation of Thistlethwaites original algorithm in which the first and third stages are skipped out. Going straight from G0 to G2 and then to G4. Kociemba essentially found a subset (A set within a set) of G, called H containing 20 billion positions. By finding an optimal solution for each of these 20 billion positions he showed that any of them could be solved in at most 18 moves, and also that any position in H could be reached by any position in G in at most 12 moves, which results in any position in G being solved in at most 30 moves. It was then later shown that 29 and 30 move cases can be avoided setting the upper bound at 28.

By March 2008 a new face had popped up in the race to find gods number, Thomas Rokicki, an American computer programmer. Using a variation of Kociembas algorithm he managed to get the upper bound down to 25, this was simply done by solving a smarter set of positions optimally. This is done by removing a large number of positions from play , showing that they only occur in conjunction with other positions.

The final goal of this whole endeavor is being able to solve every single of the roughly 4 quintillion positions to definitively show gods number, the problem with this goal is the shear amount of computing time it would take to solve every position is phenomenal. Since Thistlethwaites original algorithm the only advancements have been simplify the number that needs to be solved more and more , but what you lose when you simplify is accuracy which is why even though more and more positions are being solved we are still not at the true gods number.

Whilst the search for the elusive upper bound is going on the race is also still on to find the lower bound, a much simpler problem as you only need to find one move which cannot be solved in a certain number to set that number as the lower bound. When thistlethwaite first started he showed that the lower bound was 18, it wasn’t until 14 years later when Michael Reid set the upper bound at 28 that he also proved the position now known as ‘superflip’ to take 20 moves, giving us our final lower bound. Superflip is quite a funny position, it is where all the corners are solved, and each edge is in its right place but is flipped. You can try it yourself using the algorithm below.

**U R2 F B R B2 R U2 L B2 R U' D' R2 F R' L B2 U2 F2**

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By the end of 2008 Rokicki had managed to get the upper bound down to 22 moves, with the latest lower bound this meant that gods number had to be either 20,21 or 22.

In 2010 Rokicki and a team of other cube solvers (comprising Herbet Kociemba, Morley Davidson and John Dethridge) interested in the program managed to get exactly what they needed, the computing power that would allow them to solve every necessary position to put an end to the race for gods number. That help they got came in the form of a computing giant, Google. Distributing the positions they needed to solve across a large number of computers at Google the team were able to get through them all in just a few weeks. Google wont release information on their computer systems but for a very good home system it would’ve taken 35 years to complete all the calculations necessary to prove that gods number was indeed 20, and so that was it, for 15 years the people working on this had known that some positions required 20 moves to solve but they had finally shown that no position requires anymore.